

Statistical Methods for Population Health

Week 1: Introduction to Statistics

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Welcome!

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- **Core skills**
 - Statistical principles
 - Result interpretation
 - Basic data analysis using R
 - Some modeling techniques

Weekly Schedule

- Week 1: R Introduction and Statistical Principles

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- Week 2: Testing Mean Differences and Associations

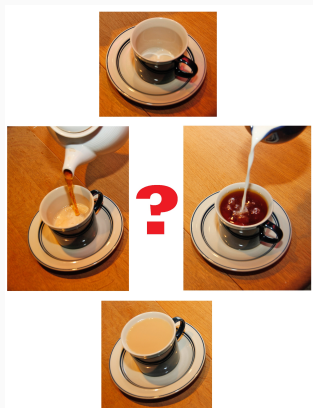
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- Week 1: R Introduction and Statistical Principles
- Week 2: Testing Mean Differences and Associations
- Week 3: Statistical Models for Multivariate Analysis

The Lady Tasting Tea

The Lady Tasting Tea Problem

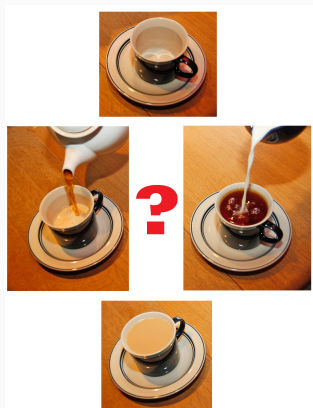
- In 1920s Cambridge, England, a Lady, named Muriel Bristol, claimed to be able to tell **whether the tea or the milk was added first** by the taste of it!



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The Lady Tasting Tea Problem

- In 1920s Cambridge, England, a Lady, named Muriel Bristol, claimed to be able to tell **whether the tea or the milk was added first** by the taste of it!
- A statistician Ronald Fisher what to test if thats true or not using **probability principles**



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- Let's **prepare many cups of tea** for her to identify, then we would expect her to identify, **on average, half** of them correctly.
- However, if she can identify **many** of them correctly, then we may have to **reject the assumption of random guessing**
- The question is, **how many is too many?**

- Two important concepts:
 1. Experimental design
 2. Hypothesis testing



Sir Ronald A. Fisher
(1890 - 1962)

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- What can be considered as “surprising” evidence given the assumption that she is randomly guessing?

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- How many cups Lady Bristol identified correctly?

Recap

Some key steps in hypothesis testing:

1). Form **Null and Alternative** hypotheses:

Null H_0 : Random Guessing vs. **Alt.** H_1 : Not Random Guessing

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- 2). Perform an experiment and **observe** that the lady identified the 4 correctly.
- 3). **If the Null hypothesis is correct**, there is only 1.4% chance that one can guess 4 correctly
- 4). This is a “small probability event” (smaller than a pre-determined **significance level**, $\alpha = 0.05$), so we will make a conclusion to **reject the Null**.

- If we reject the Null hypothesis, does it mean that Lady Bristol actually has the ability to identify them?

Correct or Wrong decision?

- We could still make a wrong decision. In fact, there are four situations:

	Accept H_0	Reject H_0
H_0 true	✓	Type I Error
H_0 false	Type II Error	✓

- Type I error:** H_0 true but we reject it.
- Type II error:** H_0 false but we accept it.

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- **Type I error** can be controlled using the α level we choose.
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- **Type II error** is difficult to analyze because we don't know what the alternative may look like. For example, the lady may have 0.7 probability to identify a correct one, or 0.9, 0.51, etc. They all can have different Type II errors.
- $1 - \text{Type II error}$ is called the power.

Summary

- Statistics is a tool to analyze data and find patterns
- However, statistics cannot provide a definitive answer
- Definitive answers come from understanding the science

Homework

- Further reading (textbook): Sections 11.3.3 and 11.3.4
 - “Quantitative methods for health research: a practical interactive guide to epidemiology and statistics” by Nigel Bruce, Daniel Pope, Debbi Stanistreet. Hoboken, NJ:Wiley, 2018 2nd edition. Wiley Online Library [[Download Link](#)]
- Install [RStudio](#) and [R](#)